U S A Mathematical Talent Search

PROBLEMS

Round 2 - Year 10 - Academic Year 1998-99

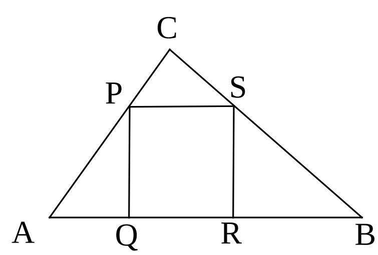
1/2/10. Determine the unique pair of real numbers (x, y) that satisfy the equation

$$(4x^2 + 6x + 4)(4y^2 - 12y + 25) = 28.$$

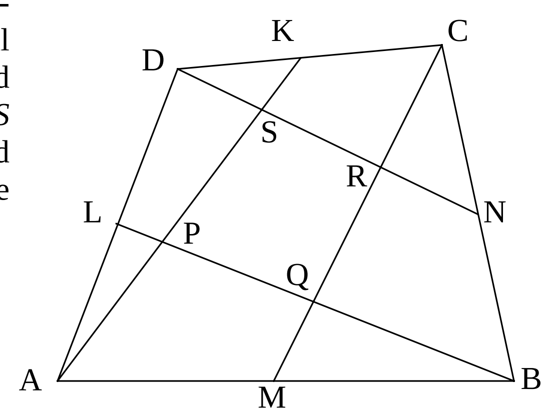
2/2/10. Prove that there are infinitely many ordered triples of positive integers (a, b, c) such that the greatest common divisor of a, b, and c is 1, and the sum $a^2b^2 + b^2c^2 + c^2a^2$ is the square of an integer.

3/2/10. Nine cards can be numbered using positive half-integers (1/2, 1, 3/2, 2, 5/2, ...) so that the sum of the numbers on a randomly chosen pair of cards gives an integer from 2 to 12 with the same frequency of occurrence as rolling that sum on two standard dice. What are the numbers on the nine cards and how often does each number appear on the cards?

4/2/10. As shown on the figure, square *PQRS* is inscribed in right triangle *ABC*, whose right angle is at *C*, so that *S* and *P* are on sides *BC* and *CA*, respectively, while *Q* and *R* are on side *AB*. Prove that $AB \geq 3QR$ and determine when equality occurs.



5/2/10. In the figure on the right, *ABCD* is a convex quadrilateral, *K*, *L*, *M*, and *N* are the midpoints of its sides, and *PQRS* is the quadrilateral formed by the intersections of *AK*, *BL*, *CM*, and *DN*. Determine the area of quadrilateral *PQRS* if the area of quadrilateral *ABCD* is 3000, and the areas of quadrilaterals *AMQP* and *CKSR* are 513 and 388, respectively.



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Complete, well-written solutions to **at least two** of the problems above, accompanied by a completed Cover Sheet, should be sent to the following address and **postmarked no later than November 14, 1998**.

USA Mathematical Talent Search
COMAP Inc., Suite 210
57 Bedford Street
Lexington, MA 02173